Prefix, Number and Name of Course: ACM 650 Random Walks and Brownian Motion

Credit Hours: 1
In Class Instructional Hours: 1 **Labs:** 0 **Field Work:** 0

Catalog Description:
Prerequisite: MAT 202 or equivalent
Symmetric random walks, ballot theorem, returns to origin and arcsine laws; gambler's ruin; Brownian motion, conditional distributions, hitting times and maxima.

Reasons for revision:

To create a one-credit-hour module for the graduate Professional Applied and Computational Mathematics program.

Student Learning Outcomes: Students will:	**Content Reference:**	**Assessment:**
1. develop and analyze picturesque theoretical models of real-world random phenomena.	I, II, III, V	Group work in class, individual homework assignments, examinations, and computer projects.
2. explain and examine the unexpected and startling properties of the symmetric random walk.	II, IV	Group work in class, individual homework assignments, examinations, and computer projects.
3. create computer simulations to calculate important features of random walks and Brownian motion.	II, III, V, VI	Group work in class, individual homework assignments, examinations, and computer projects.

Course Content:

I. Definitions and basic results
   A. Discrete-time, unit-step random walks
   B. Geometric path interpretations
   C. Restricted and unrestricted random walks
   D. Absorbing and reflecting barriers
   E. Examples (e.g. gambler's ruin)
   F. Stirling's formula and normal approximation
   G. Probability generating functions

II. Symmetric random walks
   A. Reflection principle and the ballot theorem
   B. First return to the origin
   C. Arcsine law for last returns to the origin
   D. Arcsine law for sojourn times
   E. First passage times

III. General random walks and the gambler's ruin problem
   A. Absorption probabilities

B. Mean duration to absorption

IV. Random walks in the plane and space
A. Return-to-origin probabilities

V. Brownian motion
A. Limiting value of a symmetric random walk
B. Conditional distributions given earlier values
C. Conditional distributions given later values
D. Hitting times
E. Maximum value prior to a given time
F. Gambler's ruin in continuous time
G. Brownian motion with drift

VI. Calculating with technology
A. Simulation of a random walk (generating random numbers)
B. Properties from simulated walks (histograms)
C. Evaluation of theoretical results (calculating formula values)

Resources:

Scholarships in the Field:

Barber, M., *Random and Restricted Walks: Theory and Applications*, Routledge, 1970.

Berg, H., *Random Walks in Biology*, revised edition, Princeton University Press, 1993.

Block, L., *Random Walk*, Backinprint, 2000.

Doyle, P. and Snell, J., *Random Walks and Electrical Networks*, Mathematical Association of America, 1984.

Feller, W., *An Introduction to Probability Theory and Its Applications*, Volume 1, 3rd edition, Wiley, 1968.

Freedman, D., *Brownian Motion and Diffusion*, Holden-Day, 1971.

Guillotin-Plantard, N. and Schott, R., *Dynamic Random Walks: Theory and Applications*, Elsevier, 2006.

Hughes, B., *Random Walks and Random Environments*, Volume 1, Oxford University Press, 1995.

Karatzas, I. and Shreve, S., *Brownian Motion and Stochastic Calculus*, 2nd edition, Springer, 2008.

Knight, F., *Essentials of Brownian Motion and Diffusion*, American Mathematical Society, 1981.

Malkiel, B., *A Random Walk Down Wall Street*, revised and updated edition, Norton, 2007.

Nelson, E., *Dynamical Theories of Brownian Motion*, Princeton University Press, 1967.

Revesz, P., *Random Walk in Random and Non-random Environments*, 2nd edition, World Scientific Publishing Company, 2005.

Ross, S., *Introduction to Probability Models*, 9th edition, Academic Press, 2007.

Rudnick, J. and Gaspari, G., *Elements of the Random Walk: An Introduction for Advanced Students and Researchers*, Cambridge University Press, 2004.

Spitzer, F., *Principles of Random Walk*, 2nd edition, Springer, 2008.

Telcs, A., *The Art of Random Walks (Lecture Notes in Mathematics)*, Springer, 2006.

Wiersema, U., *Brownian Motion Calculus*, Wiley, 2008.

Wilcke, W., *Random Walk*, BookSurge Publishing, 2008.

Woess, W., *Random Walks on Infinite Graphs and Groups*, Cambridge University Press, 2008.

Periodicals:

Advances in Applied Probability
Annals of Applied Probability, The
Annals of Probability, The
Applied Stochastic Models and Data Analysis
Bernoulli
Chance
Electronic Journal of Probability
Journal of Applied Probability
Methodology and Computing in Applied Probability
Probability in the Engineering and Informational Sciences
Probability Theory and Related Fields
Scandinavian Actuarial Journal
Stochastic Analysis and Applications
Stochastic Processes and Their Applications
Theory of Probability and Its Applications
Theory of Stochastic Processes

Electronic and/or Audiovisual Resources:

Electronic Journal of Statistics

Link to electronic journals web site (http://www.e-journals.org/)